

Abating Gerrymandering by Mandating Fairness

Gerdus Benadè,^{1*} and Ariel D. Procaccia²

¹Boston University, 595 Commonwealth Avenue, Boston, MA 02215, USA

²Harvard University, 33 Oxford Street, Cambridge, MA 02138, USA

*To whom correspondence should be addressed; E-mail: benade@bu.edu.

Political redistricting has been at the center of a rancorous public and legal debate over voting rights and partisanship in the U.S. Even in cases where there is a desire to craft districtings that are acceptable to both sides of the aisle, it is unclear how to do so. Our proposed approach to this problem combines fair division and optimization; at its heart is a rigorous notion of fairness for districtings, which we call the fair coin flip guarantee. We apply our approach to district four U.S. states, and find that enforcing fairness does not come at a significant cost to traditional measures of quality.

Introduction

To be elected to the U.S. House of Representatives, a candidate must win a plurality election in her district. These districts are redrawn every decade based on the most recent census; the composition and creation of districts are governed by both federal and state laws. At the federal level the Voting Rights Act requires that districts be drawn to allow minority groups to fully participate in the democratic process. Locally, many states expect districts to be contiguous and several require districts to be compact and respect ‘communities of interest.’

These guidelines, however, are often open to interpretation. For example, only six states specify a metric by which compactness is measured; elsewhere the determination of whether or not a district is compact is based on rules of thumb. Gerrymandering is the process of exploiting this flexibility by carefully drawing district boundaries for political gain, for example to protect an incumbent or to benefit (or suppress) a specific class, race or political party.

Gerrymandering has a long history. The term dates back to then-Governor of Massachusetts Elbridge Gerry's 1812 approval of a salamander-shaped district that was thought to aid his Democratic-Republican Party. In 1962, the U.S. Supreme Court ruled that population inequality in redistricting is justiciable, in part because there exist 'judicially discoverable and manageable standards for resolving it' (1). The Supreme Court has also ruled against racial gerrymandering, for example in 1960 (2) and in 1993 (3). By contrast, the Supreme Court has found it much harder to provide clear guidance around partisan gerrymandering — the difficulty is, as Justice Kennedy put it, in 'providing a standard for deciding how much partisan dominance is too much' (4).

In recent years, mathematicians have been working to address this challenge. Perhaps the most prominent approach for detecting partisan gerrymanders is based on Markov Chain Monte Carlo (MCMC) methods (5–8), which simulate a distribution over 'random' districtings. A specific districting may now be evaluated in light of this distributional information, as, in principle, significant outliers in terms of the number of seats won by one party are more likely to have been crafted for partisan advantage.

While this approach failed to sway a majority of Supreme Court justices in their 2019 ruling on *Rucho vs. Common Cause* (9), it has played a key role in the decisions made by courts in Pennsylvania (10) and North Carolina (11) to strike down the Congressional maps in these states. Eight other states have taken measures intended to prevent partisan gerrymandering, by establishing independent redistricting commissions that typically include an equal number of

Democratic and Republican members, as well as unaffiliated voters.

Regardless of the political or legal mechanism used to create them, there is a great need for rigorous methods for designing unbiased, or fair, districtings which would be acceptable to both major political parties. This problem, which is related to, but distinct from, that of identifying partisan bias in a given map, is our focus in this paper.

Designing Fair Districtings

Redistricting is often approached from an optimization perspective (12–16). This involves setting an objective — such as compactness, or the number of ‘competitive’ districts — and finding the optimal districting which satisfies various geographic and demographic constraints like contiguity and population equality. However, this approach does not necessarily lead to fair outcomes.

The prevailing consensus among social scientists is that fairness in this context is closely tied to the concept of *partisan symmetry* (17–19). Partisan symmetry ensures anonymity by requiring that parties are treated identically in the sense that each party would win the same number of seats as the other when they receive any particular fraction of the vote. To determine whether a districting in which one party wins 65% of the seats with 53% of the votes is impartial according to partisan symmetry, we must evaluate the number of seats the other party would have won had they received 53% of the votes; indeed, this comparison must be done for the entire spectrum of potential outcomes. These hypothetical outcomes are typically generated by starting from a real election outcome (or a combination of several) and applying uniform (20) or approximately uniform swings (21, 22) to model changes in voters’ political preferences. Practically, uniform swings do not allow for the types of changes in voter preferences that occur in reality, and requiring partisan symmetry under more general models of electoral systems can be infeasible. More generally, the counterfactuals required by partisan symmetry are objection-

able; for example, a Massachusetts that is 70% Republican instead of 70% Democratic is so different from the one we know that it is virtually impossible to predict how many electoral seats the Republican party would win in that alternative reality.

Instead we draw on another scientific discipline, fair division (23, 24), which deals with formal notions of fairness and ways of realizing them. One of the paradigmatic problems of fair division is that of cake cutting (25); it turns out that dividing a state between two parties is not unlike splitting a cake between two bickersome children. This observation has inspired several papers that put forward protocols by which the parties take turns splitting the state and choosing pieces (26–28). Although this approach provides fairness guarantees in abstract models, in our view its main shortcoming is that any specific, predetermined protocol cannot accommodate the myriad considerations that arise in redistricting. For example, an objection against the I-Cut-You-Freeze protocol of Pegden et al. (27) is that it generates districts that are not competitive.

Our approach is fundamentally different, in that it combines fair division and optimization, and, in a sense, enjoys the best of both worlds. On a high level, we wish to enforce an intuitive yet rigorous notion of fairness that is also binary, in the sense that it either is or is not satisfied—there is no question of degree. Among all valid districtings that satisfy that fairness notion, we find one that optimizes a given objective function. This approach—optimizing an objective function subject to a binary fairness guarantee—is akin to recent practical success stories in fair division, such as our design of a rent division algorithm (29) that has been used to solve more than 50 000 real-world instances (as of January 2020) through the not-for-profit website Spliddit.org.

To motivate our notion of fairness, imagine a procedure in which a fair coin is flipped, and whichever party wins the coin flip is given absolute power to redistrict a state as they wish (subject to the relevant laws regarding contiguity, population equality *etc.*). This procedure would lead to extremely partisan districtings *ex post*, that is, after the coin is flipped. However,

it is certainly impartial *ex ante* (before the coin is flipped), as every party is equally likely to suffer or benefit from it.

Our notion of fairness — the *fair coin flip (FCF) guarantee* — distills the essence of what makes this procedure fair, while avoiding its extreme partisan outcomes. The FCF guarantee of each party is the expected number of districts it would win under the above procedure, rounded down. In other words, it is the average, rounded down, of the maximum number of districts the party would win under any districting that satisfies the legal constraints, and the minimum number of districts the party would win under any such districting. We say that a districting is an *FCF districting* if the number of districts each party wins is at least its FCF guarantee.

A few comments are in order. First, when we say ‘would win,’ we mean under a particular distribution of voters that comes from past election data; we will come back to this later. Second, rounding is necessary, since it is impossible to guarantee that two parties each win, say, at least 4.5 districts out of nine. Third, we emphasize that an FCF districting is not obtained by flipping a coin and giving absolute power to one party; rather, this hypothetical procedure motivates a binary fairness constraint on valid districtings. Fourth, a strength of this notion is that it provides a layer of abstraction with flexibility to incorporate any future constitutional or structural changes to how districtings are drawn: any new laws or criteria that influence the districting process will change the set of valid districtings, but the definition and interpretation of the FCF guarantee will be unaffected.

In the absence of geographic constraints, each party’s FCF guarantee (before rounding) essentially reduces to their proportional share of the districts, that is, a number of seats that is proportional to their statewide support. Whatever deviation there is from exact proportionality shrinks as the number of people in a district increases. This deviation is minuscule at the scale of practical districting problems: in Pennsylvania each party’s FCF guarantee differs from their proportional share by less than 0.001%. This close connection with proportionality allows a

second interpretation of the FCF guarantee as proportionality, to the extent that is possible given the voter distribution in a state.

For example, the Republican party won roughly 32% of the Massachusetts statewide vote in the 2016 presidential election. Strict proportionality suggests that Republicans should similarly win three (roughly 32%) of the nine available congressional seats. However, this is impossible: there is no districting that complies with Massachusetts' redistricting laws under which the Republican party can win any congressional seats based on this election data (30), as the distribution of Republican-leaning voters across the state is rather homogeneous. In other words, proportionality is not a feasible standard (31). This is not necessarily disturbing in and of itself, because Supreme Court rulings 'clearly foreclose any claim that the Constitution requires proportional representation' (32). Our notion of fairness, by contrast, easily avoids this obstacle: the FCF guarantee of the Republican party would be zero (since the maximum number of seats it can possibly win is zero).

As another example, the Pennsylvania Supreme Court struck down the state's 2010 redistricting as unconstitutional and replaced it with a remedial plan (10). The political poll aggregation website FiveThirtyEight (<https://fivethirtyeight.com>) published an 'Atlas of Redistricting' in which they study redistricting across the United States. Part of this effort involved constructing gerrymandered districtings that favor either of the major political parties. Taking these districtings as the most extreme outcomes and evaluating on the presidential election data from 2016, we find that the pro-Democratic map leads to nine Democratic congressional seats (out of 18) while the pro-Republican map leads to five Democratic seats. Based on this we determine that the FCF guarantee of the Democratic party (the average of their extreme outcomes) is seven.

As intuitively appealing as the FCF guarantee is, it would not be practical if, like proportionality, it could not be enforced. We next argue, therefore, that it is always possible to find

FCF districtings in practice.

Observe that whenever it is possible to transition ‘smoothly’ between a party’s most extreme feasible districtings, it is possible to find an FCF districting. Slightly more formally, let a and b be the smallest and largest number of districts a party can win in any feasible districting, respectively. If, for every integer z satisfying $a \leq z \leq b$, there exists a districting in which the party wins z districts, then there must exist a districting in which both parties simultaneously achieve their FCF guarantees.

All of the real-world instances we have examined exhibit this smoothness, and we believe it is extremely likely to hold in practice since districts are often specified down to a census block level to ensure population equality. It is also possible to guarantee the existence of FCF districtings under abstract models of districting, for example when voters are represented as points on a plane (33).

We note that smoothness also implies the feasibility of natural variants of the FCF guarantee. For example, that there exists a districting that guarantees both parties their average number of seats over all possible districtings (rather than the average of the two extremes), rounded down. The same is true if ‘average’ is replaced with ‘median.’

FCF Districtings in Practice

Having established the conceptual underpinnings of our approach, we would like to see it in action. In the spirit of the price of fairness (34), we are particularly interested in the trade-off between the FCF guarantee and optimization objectives, that is, to what degree are FCF districtings inferior to those that optimize traditional measures of quality?

A first challenge, though, is computation. State-of-the-art machinery does not support exact optimization over the entire space of feasible districtings at the scale of real-world instances. We therefore rely on heuristic evaluation. For this, we use the GerryChain software made public by

the Voting Rights Data Institute (see <http://mggg.org>) to facilitate the running of a Markov chain.

We start from a graph representation of the state in which every node represents a precinct or voter tabulation district, and its associated properties like population, area, perimeter and the number of Democratic and Republican votes cast in various elections. State transitions in the Markov chain happen through *recombination* moves which merge two adjacent districtings before splitting them again. Before a move to a new districting is accepted, it is verified that the new districting is contiguous and satisfies population equality to within 2%.

For these experiments we sample 30 000 valid districtings in each of four U.S. states: Pennsylvania, North Carolina, Georgia and Virginia (35). The shapefiles and associated election data were prepared by the Metric Geometry and Gerrymandering Group and are available at <https://github.com/mggg-states>.

At every state of the Markov chain we keep track of the number of seats won by every party, according to the votes cast in the 2016 presidential election (36). We also keep track of the following three metrics. First, the efficiency gap of the current districting (37), which measures the relative fraction of wasted votes for each party: every vote received by the minority party in a district is declared to be wasted, while every vote for the majority party beyond the bare minimum required to secure victory is wasted. Second, compactness, as measured by the Polsby-Popper (PP) score (38), which compares the area of a district with the area of a circle that has the same perimeter length. Third, the number of competitive districts, defined as the number of districts in which the majority party has less than 54% support. Note that a smaller efficiency gap is better — a threshold of 8% has been suggested (37) — while we prefer more competitive districts and a larger Polsby-Popper score.

For each of these metrics we report in Table 1 the best value observed among the sampled districtings that satisfy the FCF guarantees of both parties, as well as (in parentheses) the opti-

	Pennsylvania	Virginia	North Carolina	Georgia
# Districts	18	11	13	14
Dem. P16 (%)	49.6	52.5	48.1	47.3
Dem. FCF seats	7	7	5	4
Comp. districts	7 (7)	6 (6)	7 (8)	6 (6)
Efficiency gap (%)	6.6 (1.2)	6.6 (0*)	4.2 (0*)	13.5 (0*)
Compactness (PP)	0.23 (0.31)	0.24 (0.25)	0.25 (0.25)	0.24 (0.24)

Table 1: For each of four states, the number of Congressional districts, normalized Democratic vote share in the 2016 Presidential election (39), the Democratic FCF guarantee, and for each of three measures, the optimal value subject to FCF and, in parentheses, the optimal value without this constraint. Absolute efficiency gaps of 0* were smaller than 0.5%.

mum across all 30 000 sampled districtings. The FCF guarantees themselves are computed via the minimum and maximum number of seats won by each party across the sampled districtings.

Generally, we find that the cost of requiring FCF districtings on traditional objectives is very low. Everywhere except Pennsylvania, the cost of imposing FCF on compactness was less than 5%. Similarly, in three of the four states the number of competitive districts possible is unaffected when imposing fairness constraints; the only exception is North Carolina where it decreased the number of competitive districts by one. Georgia is the only state where we did not observe an FCF districting with an efficiency gap smaller than the suggested 8%.

Even without access to exact optimization methods, our proposed paradigm of optimizing for some metric subject to a fairness constraint leads to very promising districtings. For example, Figure 1 shows the districting in North Carolina with the largest number of competitive districts (eight), along with the districting with the largest number of competitive districts subject to the FCF guarantee (seven). Both these districtings are significantly more competitive than North Carolina’s current districting, where the majority candidate in every district received at least 56.7% of the vote in the 2016 elections. Similarly, Figure 2 shows the most compact districting observed in Virginia (Polsby-Popper score 0.25) compared to the most compact districting satisfying the FCF guarantee (Polsby-Popper score 0.24). Both these districtings are



Figure 1: The North Carolina districting with the largest number of competitive districts (8, left), and a districting in which the Democratic party wins their target number of districts (5) with seven competitive districts (right).

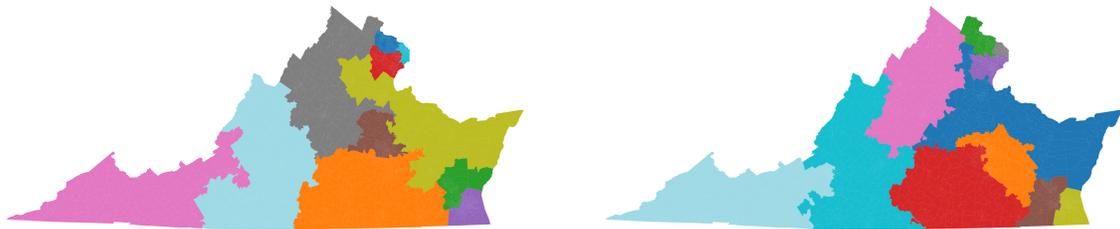


Figure 2: The most compact districting (according to Polsby-Popper) score observed in Virginia (left), in which the Democratic party wins six districts, compared to the most compact districting in which they win their FCF guarantee of 7 districts (right).

significantly more compact than Virginia’s current districting (Polsby-Popper score 0.16).

Finally, we observe that in these experiments the FCF guarantee never allows a minority to win a majority of the seats in a state.

Discussion

Our suggested districting approach relies on optimization subject to a fairness constraint. The fact that our fairness notion is readily satisfied—there are likely thousands of districtings satisfying the FCF guarantee for any state—creates the opportunity to use it in isolation should optimization-based approaches prove impossible, either because of political objections or legislative difficulties. In such cases simply requiring that districtings meet the FCF guarantee prevents the most extreme partisan outcomes yet allows legislators to retain much of the power

and freedom that comes with the ability to decide where to draw district boundaries.

A potential objection to our fairness notion is that, like partisan symmetry, it inherently relies on historical election data in order to determine what is fair, which means that partisan considerations are baked into its very definition. However, we believe that this is practical, even necessary. Indeed, even independent redistricting commissions (in all eight states that have them), which are tasked with the design of fair districtings, have a majority of members who are nominated by politicians from the two major parties. In fact, the Supreme Court has recognized that ‘politics and political considerations are inseparable from districting’ (40). While relying on historical data, our approach avoids the additional assumptions about approximately uniform changes to voter preferences that are required to generate hypothetical outcomes when evaluating partisan symmetry. Instead, we frame fairness in terms of the space of feasible outcomes for a particular election outcome and balance the claims of the two major political parties.

Another shortcoming of our approach is the issue of computation. A specific problem is that using the minimum and maximum number of seats won by a party across sampled districtings to compute its FCF guarantee does not necessarily lead to the ‘true’ value: in theory, there could be more extreme districtings that were not observed. However, this seems highly unlikely in practice. Regardless, we envision a process by which each party submits what it believes to be its best districting; the districtings submitted by the two parties can then be used to compute the FCF guarantee of each party. Under such a process, neither party would have a right to complain that the computation of the FCF guarantee disadvantaged it.

These possible limitations notwithstanding, our results show that it is possible and practical to guarantee fairness even in a climate of extreme partisanship. This is an insight that, we believe, will prove invaluable to state legislatures and independent redistricting commissions as they rethink maps that were ruled unconstitutional, and prepare for the next round of redistricting based on the 2020 census.

References

1. Baker v. Carr, 369 U.S. 186 (1962).
2. Gomillion v. Lightfoot, 364 U.S. 339 (1960).
3. Shaw v. Reno, 509 U.S. 630 (1993).
4. League of United Latin American Citizens v. Perry, 548 U.S. 399 (2006).
5. M. Duchin, *arXiv preprint arXiv:1801.02064* (2018).
6. J. Chen, J. Rodden, *Election Law Journal* **14**, 331 (2015).
7. G. Herschlag, R. Ravier, J. Mattingly, *arXiv preprint arXiv:1709.01596* (2017).
8. M. Chikina, A. Frieze, W. Pegden, *Proceedings of the National Academy of Sciences* **114**, 2860 (2017).
9. Rucho v. Common Cause, No. 18-422, 588 U.S. __ (2019).
10. League of Women Voters of Pa. v. Com. of Pa., 181 A.3d 1083 (Pa. 2018).
11. Common Cause v. Lewis, et al., 18 CVS 014001 (N. C. Super. Ct. 2019).
12. R. S. Garfinkel, G. L. Nemhauser, *Management Science* **16**, B (1970).
13. A. Mehrotra, E. L. Johnson, G. L. Nemhauser, *Management Science* **44**, 1100 (1998).
14. T. Shirabe, *Geographical Analysis* **37**, 2 (2005).
15. T. Shirabe, *Environment and Planning B: Planning and Design* **36**, 1053 (2009).
16. J. Oehrlein, J. Haunert, *Journal of Spatial Information Science* **2017**, 89 (2017).

17. B. Grofman, G. King, *Election Law Journal* **6**, 2 (2007).
18. R. D. Niemi, J. Deegan, *American Political Science Review* **72**, 1304 (1978).
19. S. Jackman, *British Journal of Political Science* **24**, 319 (1994).
20. D. E. Butler, *The British General Election of 1951* (Macmillan, 1952).
21. G. King, *American Journal of Political Science* **33**, 787 (1989).
22. A. Gelman, G. King, *American Journal of Political Science* **38**, 514 (1994).
23. S. J. Brams, A. D. Taylor, *Fair Division: From Cake-Cutting to Dispute Resolution* (Cambridge University Press, 1996).
24. H. Moulin, *Fair Division and Collective Welfare* (MIT Press, 2003).
25. H. Steinhaus, *Econometrica* **16**, 101 (1948).
26. Z. Landau, O. Reid, I. Yershov, *Social Choice and Welfare* **32**, 479 (2009).
27. W. Pegden, A. D. Procaccia, D. Yu, *arXiv preprint arXiv:1710.08781* (2017).
28. S. J. Brams, *Social Science Quarterly* (2019). Forthcoming.
29. Y. Gal, M. Mash, A. D. Procaccia, Y. Zick, *Journal of the ACM* **64**, article 39 (2017).
30. M. Duchin, *et al.*, *arXiv preprint arXiv:1810.09051* (2018).
31. J. F. Nagle, *Election Law Journal* **16**, 196 (2017).
32. Davis v. Bandemer, 478 U.S. 109 (1986).

33. For example, when voters are points on a plane it is always possible to find an FCF districting which satisfies contiguity and population equality. This model ignores practical considerations like minority-majority districts and is more granular than the census blocks used in practice, yet it establishes a theoretical foundation for the existence of FCF districtings.
34. D. Bertsimas, V. F. Farias, N. Trichakis, *Operations Research* **59**, 17 (2011).
35. This relatively small number of districtings is due to the fact that we are using recombination moves. If smaller, more local moves were used to traverse the space of districtings, several million would have been required. See the technical report 'Comparison of Districting Plans for the Virginia House of Delegates' by the Metric Geometry and Gerrymandering Group (available at <https://mggg.org/VA-report.pdf>) for a discussion of the effect of different state transitions on the required chain length.
36. Note that votes cast in the presidential election do not play any role in the congressional election. However, presidential election votes are often more readily available and are considered to be good proxies for congressional votes. Indeed, voters often do not cast a congressional vote, for example in cases where the incumbent is running unopposed.
37. N. O. Stephanopoulos, E. M. McGhee, *University of Chicago Law Review* **82**, 831 (2015).
38. D. Polsby, R. Popper, *Yale Law & Policy Rev.* **9**, 301 (1991).
39. These vote shares were calculated from the numbers published in the New York Times by discarding votes for third-party candidates. In other words, a Democratic share of 49.6% in Pennsylvania should be interpreted as the Democratic candidate received 49.6% of the votes that were cast for one of Hillary Clinton (Dem.) or Donald Trump (Rep.).
40. *Gaffney v. Cummings*, 412 U.S. 735 (1973).